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Stabilization of Takagi-Sugeno Fuzzy Systems Using Fuzzy PID, PI, and PD controllers

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Abstract—In this paper, simple and systematic ways of designing stabilization fuzzy PID (Proportional-Integral-Differential), PI (Proportional-Integral), and PD (Proportional-Differential) controllers for the Takagi-Sugeno fuzzy model are proposed. The state space representations of the fuzzy PID, PI, and PD controllers are firstly presented. Then we equivalently transform the fuzzy PID (PI, or PD) control system into the fuzzy static output feedback control system. The fuzzy static output feedback controller design for the latter system can be efficiently solved via the existing numerical optimization methods with some conservatism. Consequently, the fuzzy PID (PI, or PD) controller can be derived due to the one-to-one correspondence between the fuzzy PID controller and the fuzzy static output feedback controller. A simulation example is also given to demonstrate the effectiveness of our proposed methods.

Keywords—Takagi-Sugeno (T-S) fuzzy models, fuzzy PID controllers, static output feedback control, linear matrix inequalities (LMIs)

I. INTRODUCTION (HEADING 1)

It is well known that the PID (Proportional-Integral-Derivative) controllers have been widely applied in more than 90% of the industrial control loops, because they can offer remarkable control performances at the acceptable costs [1]. In order to achieve good performances for nonlinear systems, the traditional PID controllers have been generalized to the nonlinear ones, for example, fuzzy PID controllers [2-4]. There are two main types of the fuzzy PID controllers, i.e., Mamdani type and Takagi-Sugeno (T-S) type. A Mamdani fuzzy controller is characterized by a set of fuzzy rules, which are constructed by linguistic terms in both the antecedent and consequent parts of the fuzzy rules. In contrast, the consequent part of a T-S fuzzy rule becomes an analytical function of the premise variables. Frankly speaking, there is no essential difference between these two types of fuzzy controllers from the control perspective. They tune the coefficients of the PID controller according to different mechanisms.

The design approaches of the fuzzy PID controllers have been extensively developed and explored in the existing

literature. In [5], a type of the Mamdani fuzzy PID controller is designed based on the conventional linear PID controller, and the Bounded-Input-Bounded-Output (BIBO) stability analysis of the closed-loop system is given by the small gain theorem. The describing function method is also used to analyze the stability of Mamdani fuzzy PD (Proportional-Differential) and PI (Proportional-Integral) control systems [6]. In [7], a self-tuning mechanism is proposed for the Mamdani fuzzy PI and PD controllers to resist sudden load disturbance. Comparisons between the proposed fuzzy PI controllers and Ziegler-Nichols-tuned PID controllers have shown that the former outperforms the latter with regard to various performance indexes, such as overshoot, settling time, and integral absolute error. The Genetic Algorithms (GAs)-based Mamdani fuzzy PI+PD controller is used to eliminate the overshoot of a non-minimum phase system in [8], where the fuzzy PI controller is implemented to cancel the effect of unstable zeros, and the fuzzy PD controller is applied to reform the transient response. It is reported in [9] that a Mamdani fuzzy PI-like (fuzzy PD controller plus an integrator) controller can be effectively used in the milling process, and the stability is guaranteed by the circle criterion. Parallel to the Mamdani fuzzy PID control systems, the stability analysis and controller design of the T-S fuzzy PID control systems have been investigated as well. Based on the circle criterion, the stability conditions for the T-S fuzzy proportional and fuzzy PI control systems are proposed and explored in [10] and [11], respectively. The analytical structures of the T-S fuzzy PI and fuzzy PD controllers are examined in [12], and the BIBO stability condition is given on the basis of the small gain theorem. In [13], the conventional PI controller is used to stabilize the T-S fuzzy model. The coefficients of the controller can be derived via LMIs (Linear Matrix Inequalities). In [14], the describing function method is used to analyze the stability of the fuzzy PI and PD control systems. Both the Mamdani and T-S fuzzy PID controllers are deployed in the permanent magnet synchronous motor drive control systems in order to improve the transient response and enhance the disturbance rejection ability [15-18]. Although significant efforts have been devoted to the design of the fuzzy PID, PI, and PD controllers, there are still not enough rigorous

theoretical methods for the fuzzy controller design, i.e., how to design fuzzy PID, PI, and PD controllers for nonlinear plants with assured stability.

In this paper, we focus on how to systematically design the fuzzy PID, PI, and PD controllers for the T-S fuzzy model based nonlinear systems with parameter uncertainty. The state space representations of the fuzzy PID, PI, and PD controllers are firstly presented. Next, the fuzzy PID (PI, PD) control system is equivalently transformed the fuzzy static output feedback control system. Since the relationship between the fuzzy PID (PI, PD) controller and the fuzzy static output feedback controller is one-to-one correspondence, the design of fuzzy PID controller equals to the problem of designing fuzzy static output feedback controller. The latter problem can be efficiently handled using some existing numerical optimization methods [19-21].

The rest of this paper is organized as follows. Section 2 introduces the T-S fuzzy model, the fuzzy PID, PI, and PD controllers and their state space representations. The design of the fuzzy PID, PI, and PD controllers for the T-S fuzzy control systems is developed in Section 3. In Section 4, a numerical simulation example is provided to demonstrate the efficiency of the proposed methods. Finally, some remarks and conclusions are drawn in Section 5.

Notations: n is used to denote the n -dimensional Euclidean space. $^{m \times n}$ denotes the set of $m \times n$ matrices. In the symmetric matrices, we use $*$ to represent the terms induced by symmetry. $\text{He}\{\mathbf{M}\}$ is the shorthand notation for $\mathbf{M} + \mathbf{M}^T$.

II. PRELIMINARIES

A. T-S fuzzy model

Assume that the nonlinear plant to be stabilized can be approximated by the following T-S fuzzy model (1), which consists of r fuzzy logic rules.

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_\mu \mathbf{x}(t) + \mathbf{B}_\mu \mathbf{u}(t), \\ \mathbf{y}(t) = \mathbf{C} \mathbf{x}(t). \end{cases} \quad (1)$$

where $\mathbf{x}(t) \in ^n$ is the state vector; $\mathbf{u}(t)$ and $\mathbf{y}(t)$ are the input and output, respectively; $\mathbf{A}_\mu = \sum_{i=1}^r \mu_i \mathbf{A}_i$, $\mathbf{B}_\mu = \sum_{i=1}^r \mu_i \mathbf{B}_i$, and $\mathbf{C} \in ^{n_y \times n}$; $\mathbf{A}_i \in ^{n \times n}$ and $\mathbf{B}_i \in ^{n \times n_u}$ are the system matrices of the i -th local model; $\mu_i: \xi(t) \rightarrow [0, 1]$ is the membership function, and $\xi(t) \in ^p$ is the premise variable vector. Note that $0 \leq \mu_i \leq 1$ and $\sum_{i=1}^r \mu_i = 1$. **It is assumed throughout this paper that \mathbf{C} is of full row rank.**

B. State space representations of fuzzy PID, PI, and PD controllers

In this paper, the fuzzy PID controller firstly used to stabilize the above T-S fuzzy model. The PDC (Parallel

Distributed Compensation) technique [22] is used to construct the fuzzy controller. Therefore, the membership functions of the fuzzy controller are exactly the same as that of the T-S fuzzy model. The output of the fuzzy PID controller is as follows:

$$\mathbf{u}(t) = \sum_{j=1}^r \mu_j \mathbf{u}_j(t),$$

where $\mathbf{u}_j(t)$ is the output of each local controller. Each local linear controller can be described in the frequency domain as follows:

$$\mathbf{u}_j(s) = \mathbf{K}_j^p + \frac{\mathbf{K}_j^i}{s} + \frac{\mathbf{K}_j^d s}{\tau s + 1} = \frac{\mathbf{Q}_j^2 s^2 + \mathbf{Q}_j^1 s + \mathbf{Q}_j^0}{\tau s^2 + s}, \quad (2)$$

where s is the complex number frequency; τ is a known positive scalar; $\mathbf{K}_j^p, \mathbf{K}_j^i, \mathbf{K}_j^d \in ^{n_u \times n_y}$ are coefficients of the proportional, integral, and differential terms, respectively; matrices $\mathbf{Q}_j^2 = \mathbf{K}_j^d + \tau \mathbf{K}_j^p$, $\mathbf{Q}_j^1 = \mathbf{K}_j^p + \tau \mathbf{K}_j^i$, and $\mathbf{Q}_j^0 = \mathbf{K}_j^i$.

Without difficult computation, it can be seen that a state space representation of (2) is as follows:

$$\begin{aligned} \begin{bmatrix} \dot{\mathbf{x}}_c(t) \\ \mathbf{u}_j(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{A}^{pid} & \mathbf{B}^{pid} \\ [\tau^{-1} \mathbf{Q}_j^1 - \tau^{-2} \mathbf{Q}_j^2 & \tau^{-1} \mathbf{Q}_j^0] \mathbf{T} & \tau^{-1} \mathbf{Q}_j^2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{y}(t) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A}^{pid} & \mathbf{B}^{pid} \\ \mathbf{C}^{pid} & \mathbf{D}^{pid} \end{bmatrix} \begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{y}(t) \end{bmatrix}, \end{aligned}$$

where $\mathbf{x}_c(t)$ is the state of the local controller, and

$$\begin{aligned} \mathbf{A}^{pid} &= \text{diag} \left\{ \begin{bmatrix} -\tau^{-1} & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -\tau^{-1} & 0 \\ 1 & 0 \end{bmatrix}, \dots, \begin{bmatrix} -\tau^{-1} & 0 \\ 1 & 0 \end{bmatrix} \right\}, \\ & \quad n_y \\ \mathbf{B}^{pid} &= \text{diag} \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}, \\ & \quad n_y \\ \mathbf{T} &= \begin{bmatrix} [1 & 0] & & \\ & [1 & 0] & & \\ [0 & 1] & & \\ & & [0 & 1] \end{bmatrix}, \\ & \quad n_y \end{aligned}$$

In other words, \mathbf{T} is a permutation matrix such that

$$\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_{n_y} & \mathbf{b}_1 & \mathbf{b}_{n_y} \end{bmatrix} \mathbf{T} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{b}_1 & \mathbf{a}_{n_y} & \mathbf{b}_{n_y} \end{bmatrix},$$

where $\mathbf{a}_i, \mathbf{b}_i \in ^{n_y}$. Furthermore, we have

$$\begin{cases} \mathbf{K}_j^p = \tau \mathbf{C}_{j1}^{pid} + \mathbf{D}_j^{pid} - \tau^2 \mathbf{C}_{j2}^{pid}, \\ \mathbf{K}_j^i = \tau \mathbf{C}_{j2}^{pid}, \\ \mathbf{K}_j^d = \tau^3 \mathbf{C}_{j2}^{pid} - \tau^2 \mathbf{C}_{j1}^{pid}. \end{cases} \quad (3)$$

where $[\mathbf{C}_{j1}^{pid} \quad \mathbf{C}_{j2}^{pid}] = \mathbf{C}_j^{pid} \mathbf{T}^{-1}$, $\mathbf{C}_{j1}^{pid}, \mathbf{C}_{j2}^{pid} \in n_u \times n_y$.

Remark 1: It can be seen from (3) that the coefficients \mathbf{K}_j^p , \mathbf{K}_j^i , and \mathbf{K}_j^d are linearly dependent on \mathbf{C}_j^{pid} and \mathbf{D}_j^{pid} . If \mathbf{C}_j^{pid} and \mathbf{D}_j^{pid} are determined, the coefficients \mathbf{K}_j^p , \mathbf{K}_j^i , and \mathbf{K}_j^d are known consequently.

Remark 2: Although $\mathbf{x}_c(t)$ is the state of the local controller, it is also the state of the fuzzy PID controller. To see this point, note that the system matrices \mathbf{A}^{pid} and \mathbf{B}^{pid} are constant matrices, then the states of the local controllers are always same if they have same initial states.

Based on the remarks, the state space representation of the fuzzy PID controller can be written as follows:

$$\begin{aligned} \begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{u}(t) \end{bmatrix} &= \sum_{j=1}^r \mu_j \begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{u}_j(t) \end{bmatrix} = \sum_{j=1}^r \mu_j \begin{bmatrix} \mathbf{A}_j^{pid} & \mathbf{B}_j^{pid} \\ \mathbf{C}_j^{pid} & \mathbf{D}_j^{pid} \end{bmatrix} \begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{y}(t) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A}_\mu^{pid} & \mathbf{B}_\mu^{pid} \\ \mathbf{C}_\mu^{pid} & \mathbf{D}_\mu^{pid} \end{bmatrix} \begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{y}(t) \end{bmatrix}, \end{aligned} \quad (4)$$

where $\mathbf{C}_\mu^{pid} = \sum_{j=1}^r \mu_j \mathbf{C}_j^{pid}$ and $\mathbf{D}_\mu^{pid} = \sum_{j=1}^r \mu_j \mathbf{D}_j^{pid}$.

The state space representation of the fuzzy PID controller (4) can be generalized for the fuzzy PI and PD controllers in the same way. The state space representation of the fuzzy PI controller can be expressed by

$$\begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{u}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{pi} & \mathbf{B}^{pi} \\ \mathbf{C}_\mu^{pi} & \mathbf{D}_\mu^{pi} \end{bmatrix} \begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{y}(t) \end{bmatrix}, \quad (5)$$

where $\mathbf{A}^{pi} = \mathbf{0} \in n_y \times n_y$, $\mathbf{B}^{pi} = \mathbf{I} \in n_y \times n_y$, and the parameters of the fuzzy PI controllers are derived by

$$\begin{cases} \mathbf{K}_j^p = \mathbf{D}_j^{pi}, \\ \mathbf{K}_j^i = \mathbf{C}_j^{pi}. \end{cases} \quad (6)$$

Similarly, the state space representation of the fuzzy PD controller can be described by

$$\begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{u}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}^{pd} & \mathbf{B}^{pd} \\ \mathbf{C}_\mu^{pd} & \mathbf{D}_\mu^{pd} \end{bmatrix} \begin{bmatrix} \mathbf{x}_c(t) \\ \mathbf{y}(t) \end{bmatrix}, \quad (7)$$

where $\mathbf{A}^{pd} = \text{diag}\{-\tau^{-1}, \dots, -\tau^{-1}\} \in n_y \times n_y$, $\mathbf{B}^{pd} = \mathbf{I} \in n_y \times n_y$, and the coefficients of the fuzzy PD controllers are derived by

$$\begin{cases} \mathbf{K}_j^p = \mathbf{D}_j^{pd} - \tau \mathbf{C}_j^{pd}, \\ \mathbf{K}_j^d = -\tau^2 \mathbf{C}_j^{pd}. \end{cases} \quad (8)$$

III. MAIN THERETICAL ANALYSIS RESULTS

In this section, the robust fuzzy PID, PI, and PD controllers design methods will be proposed to stabilize the T-S fuzzy model (1). To achieve our objective, the stabilization of the fuzzy PID (PI, or PD) control systems is equivalently transformed into the problem of stabilizing the fuzzy static output feedback control systems. In the following analysis, the time t is dropped in the time associated variables, e.g., $\mathbf{x}(t)$ and $\mathbf{u}(t)$ is denoted as \mathbf{x} and \mathbf{u} , respectively.

A. System transformation

Defining $\mathbf{y} = [\mathbf{x}_c^T \quad \mathbf{y}^T]^T$ and $\mathbf{u}(t) = [\mathbf{x}_c^T \quad \mathbf{u}^T]^T$ as the outputs of the new T-S fuzzy model and fuzzy controller, respectively, we can rewrite (1) and (4) as follows:

$$\begin{cases} \mathbf{x} = \mathbf{A}_\mu \mathbf{x} + \mathbf{B}_\mu \mathbf{u}, \\ \mathbf{y} = \mathbf{C} \mathbf{x}, \end{cases} \quad (9)$$

where $\mathbf{x} = [\mathbf{x}^T \quad \mathbf{x}_c^T]^T$, $\mathbf{A}_\mu = \sum_{i=1}^r \mu_i \mathbf{A}_i$, $\mathbf{B}_\mu = \sum_{i=1}^r \mu_i \mathbf{B}_i$, and

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{B}_i = \begin{bmatrix} \mathbf{0} & \mathbf{B}_i \\ \mathbf{I} & \mathbf{0} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{C} & \mathbf{0} \end{bmatrix}.$$

And

$$\mathbf{u} = \mathbf{K}_\mu \mathbf{y}, \quad (10)$$

where $\mathbf{K}_\mu = \sum_{j=1}^r \mathbf{K}_j$, and

$$\mathbf{K}_j = \begin{bmatrix} \mathbf{A}_j^{pid} & \mathbf{B}_j^{pid} \\ \mathbf{C}_j^{pid} & \mathbf{D}_j^{pid} \end{bmatrix}.$$

Remark 3: Obviously, the dynamical characteristics are not changed with the transformation. Therefore, the original system and the transformed system have the same stability conditions.

B. Stability analysis

Substituting (10) into (9), the closed-loop system is obtained as follows:

$$\mathbf{x} = (\mathbf{A}_\mu + \mathbf{B}_\mu \mathbf{K}_\mu \mathbf{C}) \mathbf{x}.$$

The quadratic Lyapunov candidate function $V(\mathbf{x}) = \mathbf{x}^T \mathbf{P}^{-1} \mathbf{x}$, $\mathbf{P} > \mathbf{0}$, is used here for the analysis. The time derivative of $V(\mathbf{x})$ is

$$V(\mathbf{x}) = \mathbf{x}^T \mathbf{P}^{-1} \text{He}\{(\mathbf{A}_\mu + \mathbf{B}_\mu \mathbf{K}_\mu \mathbf{C}) \mathbf{P}\} \mathbf{P}^{-1} \mathbf{x}.$$

Apparently, if

$$\text{He}\{\mathbf{A}_\mu \mathbf{P} + \mathbf{B}_\mu \mathbf{K}_\mu \mathbf{C} \mathbf{P}\} < \mathbf{0}, \quad (11)$$

the closed-loop system is asymptotically stable.

Note that inequality (11) is non-convex since there are product terms between variables \mathbf{K}_j and \mathbf{P} . To circumvent

the problem, the method introduced in [21] is employed.

Assume that $RC = CP$, and let $L_\mu = \sum_{j=1}^r \mu_j L_j$, where

$$L_j = K_j R = \begin{bmatrix} A_j^{pid} & B_j^{pid} \\ L_j^c \end{bmatrix} R, \quad (12)$$

and $L_j^c = [C_j^{pid} \ D_j^{pid}] R$, and R is a matrix with appropriate dimensions, then we can rewrite (11) as follows:

$$\sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Theta_{ij} < 0, \quad (13)$$

where

$$\Theta_{ij} = \text{He}\{A_i P + B_i L_j C\}.$$

Summarizing the above analysis and using the techniques of [23] to handle the fuzzy summation inequality (13), we have the following Theorem 1 for the stability analysis. It is noted that the above analysis is also suitable for the design of fuzzy PI and PD controllers.

Theorem 1: The fuzzy PID (PI, or PD) control system, consisting of the T-S fuzzy model (1) and the fuzzy PID (PI, or PD) controller (4) ((5), or (7)), where $\tau > 0$ is previously known, is asymptotically stable if there exist matrices $P = P^T > 0$, R , and L_j^c , such that

$$RC = CP, \quad (14)$$

$$\Theta_{ii} < 0, i = 1, 2, \dots, r. \quad (15)$$

$$\frac{1}{r-1} \Theta_{ii} + \frac{1}{2} (\Theta_{ij} + \Theta_{ji}) < 0, i \neq j, i, j = 1, 2, \dots, r. \quad (16)$$

Furthermore, let $[C_j^{pid} \ D_j^{pid}] = L_j^c R^{-1} (C_j^{pi}, D_j^{pi}, C_j^{pd}, D_j^{pd})$ can be defined in the same way), then the coefficients of the fuzzy PID (PI, or PD) controller can be derived by (3) ((6), or (8)).

Remark 4: The matrix R is always invertible because both of the matrices C and CP are of full row rank.

Remark 5: Note that there exist product terms of K_j and R in (13) (see (12)). Because A_j^{pid} and B_j^{pid} are known constant matrices, and C_j^{pid} and D_j^{pid} are variable matrices, the new variable matrix $L_j^c = [C_j^{pid} \ D_j^{pid}] R$ can be defined, which makes L_j to be linear dependent on variable matrices R and L_j^c . Conversely, R and L_j^c can be used to derive the matrices C_j^c and D_j^c .

It is worth mentioning that solving conditions (14)-(16) is actually an SDP (Semi-Definite Programming) problem. The MATLAB Toolbox—YALMIP [24] is used here, which can recognize and solve SDP problems efficiently

Corollary 1: The fuzzy PID (PI, or PD) control system, consisting of the T-S fuzzy model (1) and the fuzzy PID (PI, or

PD) controller (4) ((5), or (7)), where $\tau > 0$ is previously known, is asymptotically stable with predefined decay rate ν if there exist matrices $P = P^T > 0$, R , and L_j^c , such that (14)-(16) are satisfied, where

$$\Theta_{ij} = \text{He}\{A_i P + B_i L_j C\} + \nu P.$$

Furthermore, let $[C_j^{pid} \ D_j^{pid}] = L_j^c R^{-1} (C_j^{pi}, D_j^{pi}, C_j^{pd}, D_j^{pd})$ can be defined in the same way), then the coefficients of the fuzzy PID (PI, or PD) controller can be derived by (3) ((6), or (8)).

IV. A SIMULATION EXAMPLE

In this section, a simulation example [25] is given so as to show the effectiveness of the proposed design methods. Consider the following two-rule T-S fuzzy model:

Plant rule i : If ξ is M^i ,

$$\text{then } \begin{cases} \dot{x} = A_i x + B_i u, \\ y = Cx. \end{cases}$$

where $x = [x_1 \ x_2]^T$, $\xi = y$, and

$$A_1 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 12.6305 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -0.0779 \end{bmatrix},$$

$$C = [3.5 \ 1].$$

The membership functions are defined as follows:

$$\mu_1 = (1 - (1 + e^{-7(\xi - \pi/4)})^{-1})(1 + e^{-7(\xi + \pi/4)})^{-1},$$

$$\mu_2 = 1 - \mu_1.$$

In our example, the fuzzy PID controller (4) is first used to stabilize the T-S fuzzy model, we can find the following matrices satisfy Theorem 1 with $\tau = 0.05$.

$$P = \begin{bmatrix} 0.0431 & -0.0269 & 0.0062 & -0.0889 \\ -0.0269 & 0.1297 & 0.0018 & -0.0254 \\ 0.0062 & 0.0018 & 0.0133 & -0.0174 \\ -0.0889 & -0.0254 & -0.0174 & 1.2326 \end{bmatrix},$$

$$R = \begin{bmatrix} 0.0133 & -0.0174 & 0.0018 \\ -0.0174 & 1.2326 & -0.0254 \\ 0.0234 & -0.3366 & 0.0354 \end{bmatrix},$$

$$L_1^c = [0.5930 \ -8.4727 \ 1.3242],$$

$$L_2^c = [0.9440 \ -13.5831 \ 2.2182].$$

Furthermore, we have

$$[C_1^{pid} \ D_1^{pid}] = L_1^c R^{-1} = [-23.3382 \ 4.1609 \ 41.6127],$$

$$[C_2^{pid} \quad D_2^{pid}] = L_2^c R^{-1} = [-43.1355 \quad 7.5839 \quad 70.3510],$$

$$[C_{11}^{pid} \quad C_{12}^{pid}] = C_1^{pid} T^{-1} = [-23.3382 \quad 4.1609],$$

$$[C_{21}^{pid} \quad C_{22}^{pid}] = C_2^{pid} T^{-1} = [-43.1355 \quad 7.5839],$$

where T is a 2×2 unit matrix. From (3), we obtain

$$K_1^p = 40.4354, \quad K_1^i = 0.2080, \quad K_1^d = 0.0589,$$

$$K_2^p = 68.1752, \quad K_2^i = 0.3729, \quad K_2^d = 0.1088.$$

The responses for the initial condition $x(0) = [\pi/4 \quad 0]^T$ are depicted in Fig. 1 and Fig. 2, where we can see that the closed loop is asymptotically stable.

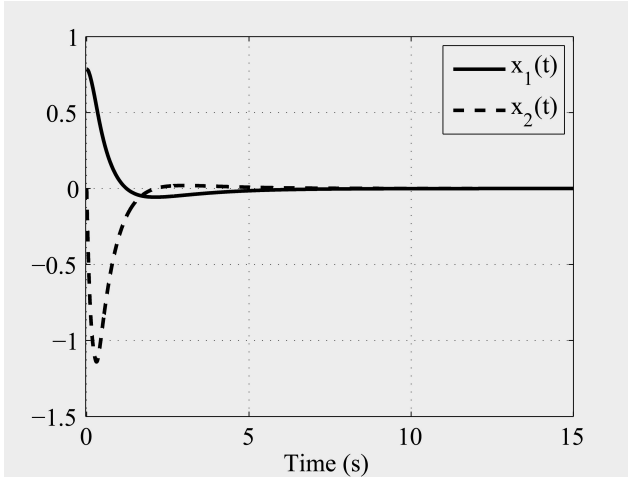


Fig. 1. Responses of $x_1(t)$ and $x_2(t)$ with initial condition $x(0) = [\pi/4 \quad 0]^T$.

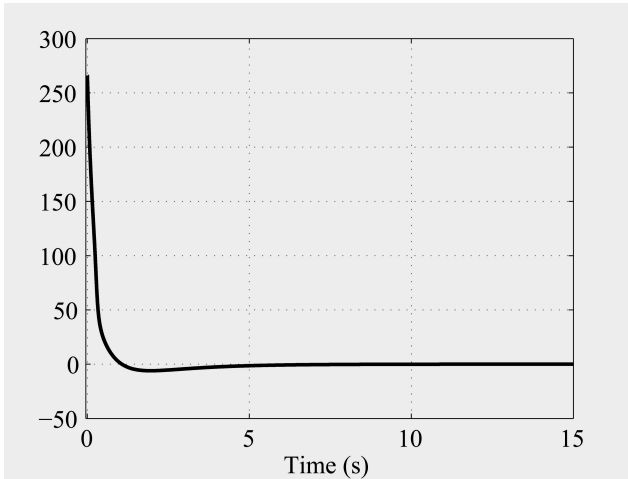


Fig. 2. The output of the fuzzy PID controller.

Similarly, the fuzzy PI and PD controllers can be used to stabilize the T-S fuzzy model. To save the space, only the design results are provided. For the fuzzy PI controller, we have

$$K_1^p = 63.7749, \quad K_1^i = 17.0167,$$

$$K_2^p = 109.7688, \quad K_2^i = 30.7108.$$

For fuzzy PD controller, we have

$$K_1^p = 41.5227, \quad K_1^d = 0.0457,$$

$$K_2^p = 69.0421, \quad K_2^d = 0.0813.$$

V. CONCLUSIONS

In this paper, simple and effective fuzzy PID, PI, and PD controllers design methods have been proposed and studied. Firstly, state space representations of the fuzzy PID, PI, and PD controllers are presented. Then the fuzzy PID (PI, or PD) controller design problem is transformed equivalently into the problem of designing the fuzzy static output feedback controller. Next, the latter problem can be efficiently handled by the convex optimization techniques. A simulation example has been finally given to demonstrate the effectiveness of our approaches.

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